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A LIGHTNING EMPIRICIST *literary supplement*

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Preliminary Edition

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Preliminary Edition

This is, in effect, a guest editorial from George A. Philbrick, who still is greatly attached to analog instruments, and whose name has been among those most closely associated with Operational Amplifiers over the years.

It is intended to be, and the subject certainly deserves, a cohesive treatment of the Nature and Applications of such Amplifiers: in a serial form thus begun.

Collaborative effort, including counter-opinion from more orthodox critics, will not be unwelcome. This might even lead, ultimately, to replacement(s) for the above writer, in case under the burden of unburdening, or from other causes, he does not survive.

In any case, subsequent texts of the proposed series will appear when they are ready.

Comments or contributions should be addressed to Editor, Lightning Empiricist, care of the Company.

On Operational Amplifiers

Part I: THE LOCAL LOOP IN THE GENERAL CASE

George A. Philbrick

Expository approaches from several directions will be required for a proper presentation of the Operational Amplifier in structure and function. Historical, philosophical, and utilitarian viewpoints will be included in due course. Meanwhile a reasonably academic, and concise yet comprehensive introduction is surely expected. Such will be the goal of this first part.

Pending more formal definition, an operational amplifier may be said to play a part which generalizes upon those of the more familiar but more rudimentary electronic "valves", namely vacuum tubes, transistors, etc. It increases the power level of signals and information, serving as the life-giving (or active) element in an operating circuit, thus enabling such a circuit to perform in a premeditated and predictable fashion, as in making *operational* calculations. Broadly speaking, these amplifiers and circuits and signals need not be electronic or even electric – although we shall shortly assume them to be so. We shall also assume them to be DC or direct-coupled, working down to zero frequency, though indeed they may not always be so applied. In regard to the signals themselves, we can at an early stage take them to embody instantaneous voltages or currents, varying in general through zero, and over positive and negative ranges with equal facility.

Why *both* voltages and currents? Much of the technical literature emphasizes voltage signals as standard operational currency; this is much too restricting, in view of modern practice. It is not simply a matter of the classical duality between current and voltage. As will become evident there is special propriety, for the operational technologist, in dealing interchangeably with these two electrical variables. In any case, whether signals are best expressed in terms of voltage or in terms of current, it is clear that all signals carry information only if attended by a flow of *power*. Thus both of the former variables are normally involved, their instantaneous product being the embodiment of instantaneous power, with the algebraic sign of the product denoting the direction of power flow, information flow, and (again, normally) of causality. For the present we must leave these profoundities; actually in the presence of strong feedbacks, an operational hallmark, there arise certain paradoxes which must be wondered at, possibly resolved, and preferably exploited.

Strictly speaking, the information submitted to operational circuitry need not be borne by signals measured as current or voltage alone, or even explicitly as power. For example an input may be reflected in the value of a varying impedance element, the "signal" amounting then to the ratio between some voltage and some current, or to various ratios among these quantities and their derivatives with respect to each other or with respect to time.

There are many other exceptions to the voltage-and-current regime in operational practice. For convenience, we can assume here that we are dealing with continuous variables of either character, as for instance in classical circuit theory, and as is generally assumed in the contemporary instrument technology. Further, we cannot pretend in this context, exalting as this might be, to be saying "operational" in the manner of Eddington or Bridgman, or even of Heaviside. We chiefly address systems engineers of the practical sort.

The Operational System

It is usual to introduce operational amplifiers in a progression of relatively simple applications. Demonstrations of such special cases admittedly have pedagogical value as well as practical importance. They will be dealt with considerably in what follows, for both these reasons among others. Here, we shall begin with a description of the most general circuit, or instrumental subsystem, involving a single operational amplifier. This will enable the framing of a meaningful definition of the operational amplifier itself, and should help to establish the scope of the doctrine to be presented. It may incidentally give some plausible notion of the versatility of such amplifiers as instrumental tools.

Shown in Figure 1.1 is an intentionally generalized operational system, having a single amplifier and a major local feedback loop. This

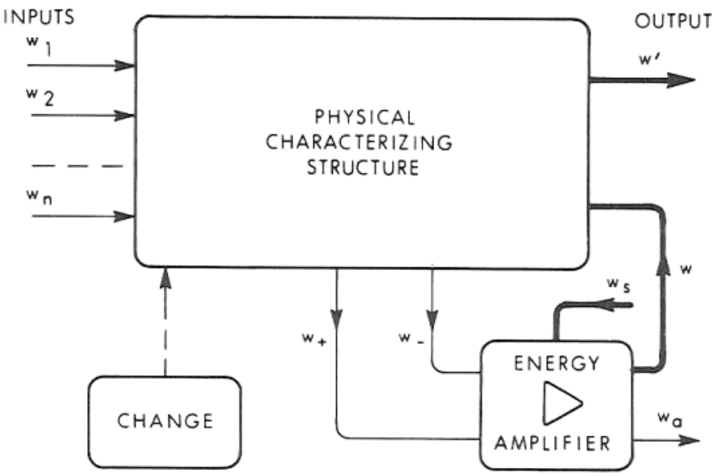

Figure 1.1
Block Diagram of a Local Operational System

is a block diagram. It applies to any physical medium or combination of media, with whatever physical variables are appropriate thereto. It shows the causal relationships among such variables, as brought about by the arrangement of the system. Although perhaps we cannot always associate energy flow with causal direction, it is nevertheless intended that each of the symbols associated with the variables of this figure signifies *power:* the rate of energy transport in the direction indicated by each arrowhead. The arrows may also be taken to denote the directions in which information and *influence* are intended to flow. Notice in passing that a closed path of action is traceable within the system, and that power levels are roughly shown by weight of the flow paths, which are not to be confused here with ducts or conductors. We can enjoy such confusions in greater detail at a later stage.

The system shown in Figure 1.1, if not further restricted, could be mechanical, thermal, hydraulic, or chemical as well as electrical, electronic, or even electromagnetic. It could also be mixtures of these forms, among others. Historically, as will be brought out later, several non-electrical forms were not unfamiliar. While it will be edifying to trace out the evolution of modern operational systems from such older forms, and to show the close analogies which obtain among the respective power-bearing signals, our concern for the most part will be with electrical apparatus and techniques. As implied a little earlier, the term "electric" is intended to include magnetic and electromagnetic as well as electronic phenomena and equipment. In particular the term "operational", especially as regards amplifiers, is today largely electric by connotation. In spite of this, precisely equivalent structures, but not so termed, have been useful in the past; and future equivalent structures in other media will most certainly be so identified. It should also be kept in mind that there are energy flows other than electric in even the most recent solid-state instruments. Thus it is seemly to hold a broad and catholic view of the variables and energy forms in the systems we discuss, and the reader is asked to maintain this generalising attitude even if this writer lapses. We proceed to a description of the system illustrated in Figure 1.1.

A local operational system is formed from two, more or less distinct, subsystems: a characterizing structure and an amplifier. These are so interconnected that each influences the other directly and reciprocally, providing thus the opportunity for action in a closed causal loop. Although inputs to the system may influence the amplifier directly in certain special cases, in general they act through the characterizing structure, traversing what may be considered its forward path. The principal output of the system traditionally coincides with the amplifier output itself, which in turn is also the sole input, from within the system, to the characterizing structure, the others being system inputs also. This traditional case follows as a routine special case from that shown in the figure, in which the principal system output is shown emerging from the characterizing structure. An additional output from the amplifier is an option having auxiliary functions, which shall be unexplained for now, but which are harmless. Finally, a pair of interior outputs from the characterizing structure provide the amplifier inputs, completing the local operational loop via the feedback path within the characterizing structure, from the output of the amplifier.

For the type of local operational system we are discussing, a major internal duty is that of maintaining as closely as possible a condition of balance between the pair of amplifier input signals, specifically between appropriate measures of signals thereby delivered to the amplifier. Such measures may be taken to be either the *potential* or the *flux* (corresponding to voltage or current in the electrical case) involved in the signal. While the signal power is contributed by both of these measures, one or the other is typically determinate for a given signal. An amplifier may thus ordinarily be thought of as responding to just one of these kinds of signal measure. In important typical cases the amplifier input signals act additively (or subtractively!) on the amplifier, or at least nearly so; the desired balance between the aforesaid potentials or fluxes amounts then substantially to an equality. In a collection of important special cases, one of the two amplifier inputs may be nonexistent. Alternatively, that input may be maintained at a constant signal level, or identically at zero. In the latter case the desired balance level for the remaining signal measure amounts to a simple *null*.

The physical characterizing structure is perhaps most frequently a linear structure. It may also, however, be nonlinear, this then being the general case. For our present purposes it may be assumed to be passive: containing no sources of power, or amplifiers of energy. If it did have steady sources of power, for example, these could be regarded as inputs and so moved outside the structure. If it were to contain amplifiers, as it conceivably might in some even more enthusiastic view of Figure 1.1, we should then assume that these amplifiers were part of another local operational system, remote from though appropriately interconnected with, the system under consideration here.

Generally speaking, the (passive) characterizing structure may have been designed specifically to enable the operational system to succeed in a particular purpose, or it may have arisen from natural causes beyond the control of the designer. For the moment it is preferred to entertain both possibilities. Of course it is useful that there should exist, within this structure, valid paths whereby all the relevant feed-forward and feedback actions may take place. In any case, as implied by the name, it is evident that the dynamic or functional character of the operational system as a whole is principally determined by this characterizing structure.

Input signals to the local operational system may commonly arise from other operational systems, being outputs thereof. They may derive from fixed sources or proscribed functions of time generated externally. They may be supplied in appropriate physical form from measuring transducers. In any case they will provide *independent* variables for the local system, determined from without in terms of potential or flux, though not usually in terms of both.

The principal output of the operational system, which in many cases is that of the operational amplifier as well, is always intentionally identified and prescribed as being either a potential or a flux, but not both. While dutifully maintaining one such measure as its output signal, the amplifier simultaneously delivers power to subsequent destinations, expending it as needed to perform a desired manipulation. A typical such destination, naturally, would be the input of a separate but related operational system. Since the nature and the variability of the load to be served is not determined by the inputs of the local system, a requirement is the ability to meet the demands of a broad range of possible driven structures. This must be accomplished without interfering with the operationally prescribed output variable, which, be it potential or flux, acts thus as the *dependent* variable of the local operational system.

Finally the amplifier itself, as the subsystem of the local operational system in which our interest here resides, must be endowed with properties and qualifications which will permit it to achieve the special performance demanded of it. Of course it must have power gain in liberal amounts, from zero frequency (infinite period) to some maximum frequency (minimum period) consonant with intended applications. It naturally must be able to provide the appropriate polarity in terms of the relevant signal measures, over this range and to offer a thoughtfully specified variability, not only of the numerical gain itself, but of the phase (or time) delays. These properties must be so planned that the attenuation and phase spectra of the expected characterizing structure, acting in sequence with the amplifier, will

yield combined dynamics such that under all circumstances the loop thereby formed will be stable as well as effective in balancing the amplifier inputs and in providing the desired dependency of the system output. In addition, the amplifier must have a high signal-to-noise ratio, stability of the input thresholds (with respect to time, temperature, and the energy supply sources), adequate independence of its output in the face of difficult or varying loads, proper ranges over which both inputs and outputs may vary without saturation, and satisfactory behavior in the event of limiting or saturation.

A parenthetic word on the symbology of Figure 1.1, to which we have chosen not to refer above. The w was chosen as a power symbol for all variables or signals, representing *watts* for example, but purposely generic as to physical medium. Numerical subscripts denote inputs to the local operational system. Plus and minus subscripts denote the amplifier inputs, somewhat arbitrarily assignable. The s and a subscripts are for supply of power to the amplifier, and for an auxiliary output, respectively. The primed symbol, and the symbol alone, are for the principal system output, and for the amplifier's own output, in that order. Where these are coincident, the prime would be deleted. All power variables are considered to be instantaneous quantities. In the electrical case, each would be precisely the ei product, with the algebraic sign preserved.

The Operational Circuit

The section immediately preceding treated a local operational feedback system in general physical terms, showing that operational methods and concepts are not limited to electrical forms and variables. We shall be largely so limited in what follows, however; but the remarks of the above paragraphs should still apply.

By analogy with the local operational system, we may now define a local operational circuit as being made up of a passive characterizing network and an operational amplifier, interconnected as shown in Figure 1.2. This figure is not a block diagram, and in this respect is quite unlike Figure 1.1. Nor is it a signal flow graph. Block diagrams and signal flow graphs will both be applied by us, in what follows, for

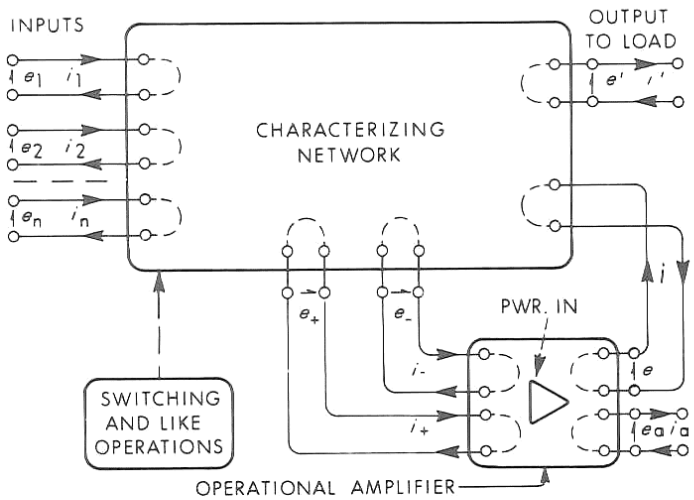

Figure 1.2
Schematic Diagram of a Local Operational Circuit

their clarity and generality. Figure 1.2. however, is just a wiring schematic, with real electrical terminals, conductors, voltages and currents. It shows the two major components of an operational circuit, namely a characterizing network and an operational amplifier, co-acting and interconnected in a feedback configuration, and interposed between a set of input signals and (at most) two output signals. All signals are presented as voltage-current couples, embodying directed power quantities or flows of energy. Each signal, considered as either voltage or current but not both, is normally associated with and identified as the prescribed variable for that signal, the other variable of the couple then being, automatically, whatever is required to supply the needed power.

Consider now the input signals themselves, which in the causal sense are the origins of all action in the circuit. Each of these signals is normally identified, individually, with a prescribed (or enforced) voltage or current. The input voltages and currents, as a set of *n*, make

up the independent variables of the local operational circuit, in dependence on which the principal output variable will behave, or *vary*. Those input variables which are prescribed as voltages may be presumed to arise from arbitrarily "stiff" sources: having zero impedances. Those prescribed as currents may be thought to derive from sources of infinite impedance. Less ideal sources are also thus representable, without loss of generality, through appropriate modification of the characterizing network. The permissible ranges of the input variables have substantially no upper limits, no matter what range the principal output variable is limited to. This is owing to the accommodating flexibility of the characterizing network *per se.* The lower limits of the input ranges, however, are dictated in effect by the stability, resolving power, and noise levels of the amplifier input(s).

Naturally the input signal sources need not be "floating", as they are made to appear in Figure 1.2, although we take this to be the general case. Similar remarks are valid, in fact, for all the signals shown in this figure. In a common special case the input variables, being regarded (say) as voltages, may all have a terminal in common, and this same terminal may be coincident with one of the terminals of any or all of the remaining terminal pairs in the circuit. Similar special circumstances are demonstrable, although not as obvious, in the case of a set of (say) prescribed input currents. (Compare Figure 1.4 below.) And so on, through myriads of intermediate and rudimentary special cases.

As before, the input signals to a given operational circuit are quite frequently the output signals of other operational circuits. They may also arise as manually manipulated quantities, as transducer outputs, as the outputs of periodic or random signal generators, or as fixed voltage or current sources. The (maximum and average) power consumed from a current or voltage input depends, quite evidently, not only on the excursions of such an input variable, but also on that portion of the characterizing network to which the terminals apply. Fortunately for the operational craft, the ranges of impedance available for passive network elements are very broad, so that it is seldom necessary to ask for an unreasonable consumption or expenditure of power.

At the output end of the operational circuit, we may first dispose of the amplifier's auxiliary output signal. With some exceptions, the purpose of this signal is to monitor some sort of abnormal behavior within the amplifier, possibly denoting an untoward condition elsewhere in the local feedback loop. It is nothing to fret about, for now.

The other, or principal, amplifier output signal is by definition an input signal, in its turn, to the characterizing network. This output operates normally at the highest power level in the operational circuit. It must supply electrical energy not only to the characterizing network, but, more significantly, to external loads *via* the principal output of the operational circuit as a whole. This latter output is shown in Figures 1.1 and 1.2 as emerging from the characterizing network; an explanation is thus in order.

The fact is that in a large fraction of cases these two outputs coincide, at least as regards the voltage or current embodiments thereof. Conventional output circuitry for the two cases latterly implied is shown in Figure 1.3. The voltage-drive case (a) delivers the same voltage to the external load – that is to the output – as that which is returned to the characterizing network (hereinafter CN). In the other case (b) an identical current drives the load and the feedback input of the CN. In either case the power required of the operational amplifier (hereinafter OA) output is, instantaneously, the unprimed voltage-current product *ei,* and will be seen to depend not only on the character of the CN but on that of the load, which will in general be undetermined. It is of course the obligation of the OA to supply this power as required, up to some appropriate maximum level. The ultimate source of such power is considered to be implicit in, and indigenous to, the OA. While this may be literally obvious, as when an individual OA has a private set of batteries, more typically the physical power supply is separate and external, and must serve a number of OA's in tandem.

It is further considered that the OA shown in Figures 1.2 and 1.3 includes whatever auxiliary power-amplifying means may be appended at its output end. Such means are frequently packaged separately, and are called "boosters". They typically augment the range of current available from an OA without altering the attainable voltage swing, though indeed such augmenting components may increase either or

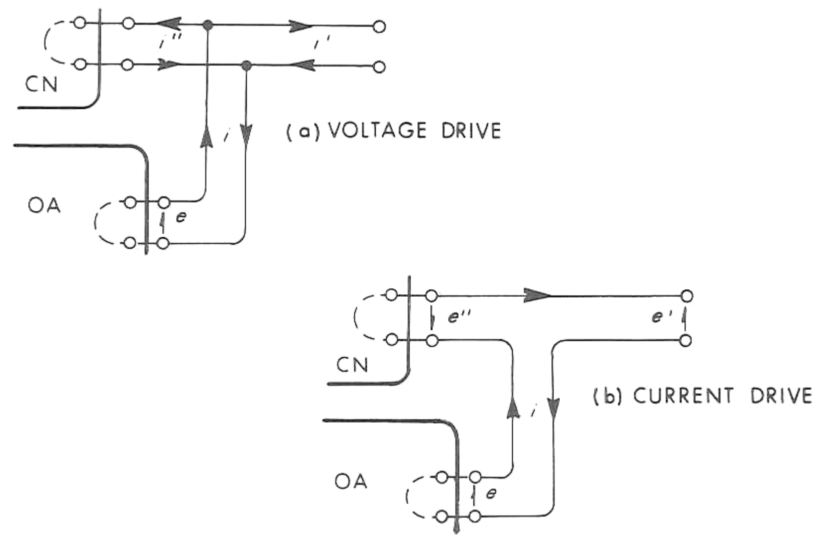

Figure 1.3
Conventional Output Configurations for Circuit of Figure 1.2

both of these capabilities. In any of these cases, the maximum power output, and usually the power gain, is thereby increased, albeit with some new risk of instability to be overcome; but in our subdivision of the operational circuit such a booster is to be lumped with the rest of the operational amplifier.

We return now to explaining the generalized form of output circuitry shown in Figure 1.2. There, unlike the more direct conventional cases of Figure 1.3, the operational circuit output is distinct from the OA output, and emerges from the CN. While the latter is ordinarily selected or designed on fairly well formulated criteria to impart preassigned mathematical performance to the whole circuit, nonetheless alterations or additions are frequently made to improve stability or effectiveness in the face of the imperfections of reality. The network elements inserted for this purpose may be regarded as belonging either to the OA or to the CN, according to whim, although the logical choice is generally clear. We are evidently making the latter choice for passive circuit additives which are interposed between the OA and the overall output of the operational circuit. Even with no such interposition, the output arrangement of Figure 1.2 must be accepted as the simplest generalisation of the two cases given in Figure 1.3.

So outputs from the operational circuit are either directly from the OA, or from a part of the CN which is in fact normally not electrically distant therefrom. If the operational output is intended as a voltage signal, it is in the nature of the circuit's accomplishments that the output impedance is made to be low, especially as compared with that of any device which it drives, or of the few or many other circuits to which it may be connected in parallel to serve as input. This is largely a consequence of the (dominantly negative) feedback path(s) to the OA input(s) through the CN, although a low impedance of the OA output in its own right is justifiably considered a virtue for the case of voltage-driving. In the seemingly opposite case, in which a prescribed current is to be driven into some load, or into more than one load arranged in series, the ideal output impedance is naturally a very high one. Here again, however, it is a virtue of the feedback loop linking OA and CN that it greatly assists in attaining the desired circuital output characteristic. We must note in passing that the output couples of two or more current-driving operational circuits may be connected in coincidence across a given load, an unthinkable blunder for the voltage-driving kinds, and that in such a connection the load current sums directly the individual output signals.

Before turning to the nulling or balancing connections from the internal CN output(s) to the OA input(s), we may first take care of instances in which an operational circuit output is connected as an input to the same circuit. These cases give no trouble, since one may simply delete that input as an independent circuital input and incorporate the connection as part of the internal feedback circuitry of the CN. There are many less simple, but equally legitimate and important cases which arise when some casual path permits an operational output more subtly to influence an input of the same operational circuit indirectly. Where the intermediary circuitry is

passive, similar treatment is normally feasible, possibly with some fairly obvious extensions. Even when such circuitry is mildly active, it may often be considered approximately passive with no great violation, although attention to reproducibility and precision of the assumed result is in general deserved. In the cases of violently active intermediary circuitry, other than operational amplifier systems, these are best treated as switching operations (considered elsewhere in this series) or as "piece-wise passive" phenomena in the time dimension. In most cases of active intermediaries which comprise OA's, these may be placed in the realm of multiple interconnected operational circuits, which have been omitted in this Part. They will be elaborated upon later, according to plan.

Completing the causal loop of the local operational circuit are the interior outputs of the CN, which act in turn as the inputs of the OA. The signals involved in these interior interconnections are in general carried from two pairs of terminals in the CN to corresponding terminals in the OA. At least this is the most general case considered here; there is always the chance of future obsolescence. Of course one pair suffices in many special cases, as do three terminals in a somewhat wider special class. In this context we have chosen our preferred degree of universality already. These signals, each in general comprising both voltage and current, carry power at the lowest level in the operational loop. In fact if the input impedances of the amplifier are either substantially zero or substantially infinite, the latter especially being a favorite ideal specification, then the power delivered to the amplifier must substantially vanish. The practice of matching impedances, incidentally, has little meaning here or elsewhere in operational circuitry.

When the OA is of the sort which responds to voltage at each input, or to current at each input, then these responses are traditionally offsetting, so that it will respond essentially to the corresponding difference in voltage or current. This is anyhow normally the intention, in a so-called Differential Operational Amplifier (DOA). Failure to respond in a balanced fashion at its inputs, in a DOA, is measured in terms of its so-called "common mode rejection" – to be defined elsewhere.

It is also conceivable that one input could be current-responsive and the other voltage-responsive. In such a case the direct difference referred to would have to be replaced by a dimensionally weighted expression, with a crucially precise conductive or resistive factor relating the two signals. For each input considered alone, the relationship between input current and input voltage is naturally determined by the OA input conductance (or admittance) or the OA input resistance (or impedance) for that input. In normal DOA practice, these properties are not required to be extraordinarily precise or stable in time. In dealing with such amplifier signals, and with such parameters as input impedance which relate them, it is advisable to distinguish between signal variations and the corresponding absolute magnitudes. These will in general differ in view of relatively steady "bias" quantities, of voltage or current, arising either within the amplifier or ahead of it, and which are either inherent or imposed to counteract unbalances of one kind or another. Generally speaking, it is wisest to deal with *changes* in such signals, and with such parameters as impedance on the basis of changes rather than total signal magnitudes; or better, to regard the change as the signal itself. More generally, whatever tactics and assumptions are made in this regard should always be stated in practice to avoid serious misunderstandings – which are almost inevitable when biases and thresholds and nonlinear characteristics are ignored. These admonishments are of course applicable everywhere, but they are critically appropriate for amplifier inputs. We offer only mild apologies for this digression.

With a single amplifier input signal, the feedback operation in the loop which contains the OA will, when successful, reduce this signal to a relatively small value and maintain it there. The amplifier input signal is thus in the nature of an *error* signal, or a variable-to-be-nulled, so that the behavior of the operational circuit is truly that of *null seeking*. With two input signals to the OA, and the desired condition an equality of sorts between them, the task of the operational loop is to seek such equality, thus striving to make one of the signals "follow" the other. In this sense we may speak of the circuit as a *follower*, as indeed many of the important operational circuits are referred to in certain special cases. Nevertheless, in a basic sense they are still null seekers with regard to the difference between the input signals.

The plus and minus signs applied as subscripts for the nulling signal links from CN to OA are, at this stage, completely arbitrary. They do have mnemonic value ultimately, and have come into popularity for DOA's to designate the algebraic sign of the relative response induced by each input at the OA output. Referring again to Figure 1.2, the internal nulling links may be reduced to the relatively familiar 3-terminal case by connecting (making coincident) the lower or left conductor of each signal pair to each other, the common conductor or terminal then serving as a reference.

In fact, a quite serviceable special case of the operational circuit of Figure 1.2 is obtained by replacing all the signal pairs involved by single connections referred to a common reference. Such a circuit, shown in Figure 1.4, suffices for some 90% or more of modern practice. Nor need it apply only to voltage signals exclusively, although such would be the most familiar. Nevertheless, we dare not abandon the generality of the former figure. In the first place there are practical cases in which a common reference is out of the question. Even more vital, however, is the conviction that the transport of signals by conductor-pairs is not only the most general conceptual scheme but is suggestive of more advanced present and future techniques.

We now look a little more carefully at the CN, and then at the OA itself.

The Characterizing Network

The function of this network in the operational circuit is complementary to that of the OA, but differs dramatically therefrom. Although there are many varieties of OA, suited to many environments of application, any given type is expected to serve unaltered when applied in any one of a countless assortment of operational circuits. The OA must co-act with the CN, in each such case, to perform a correspondingly countless assortment of operations. It is the CN then, which principally determines the operational function to be performed. Therefore also, as already implied by the considerable generality of its portrayals in Figures 1.1, 1.2, and 1.4, the CN is encountered in an even wider variety of forms.

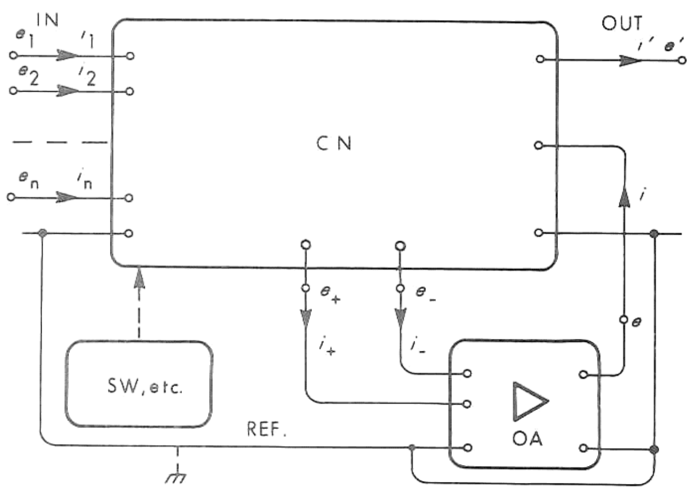

Figure 1.4
Conventional Specialization of the Operational Circuit of Figure 1.2

Some forms of CN are almost trivially simple, even with no electric circuit elements at all. Others may have grotesquely complicated collections of elements, standard and exotic. Some of the elements may be fixed and specialized, others adjustable over a broad range of characteristics. The adjustments may apply to the individual circuit elements, or they may involve switching operations which can alter the network fundamentally. Such adjustments and switchings may be manual or automatic, local or remote. The network may be based on an exact mathematical formulation, or on an approximation. The describing expressions may be entirely algebraic, or they may require differential equations with time as an independent variable.

The CN, and consequently its elements, may be linear in nature: at least by intention. In such a case the CN is conveniently describable by classical differential operators of the operational calculus, or alternatively by the complex (real and imaginary, not necessarily intricate!) impedance and admittance functions commonly applied to linear electrical networks and filters. Still linear, though in a broader

sense, are those normally linear systems in which the parameters or characteristics vary with time in a prescribed manner; networks of this kind may yield to a somewhat stronger kind of analysis. However, in the cases in which the CN, and consequently one or more of its elements, are nonlinear in character, the conventional tools of analysis and synthesis may be severely limited. The nonlinear CN, at best, requires piece-wise linear methods or graphical analysis, and at worst exhaustive and exhausting experimental procedures - which may nevertheless be worthwhile. In particular, methods involving frequency spectra for amplitude and phase must generally give way to studies in the time domain. This is not all a loss, however, since time-domain reasoning and experiment are powerful, and well adapted to modern methods of testing.

We have been expounding as though the CN were entirely independent of the OA, which of course in a practical sense it is not. The transformation, from the characteristics of the CN alone, to those which result for the whole operational circuit, is naturally a central part of the game, as we shall see. For the present, the general properties referred to above for the CN may be assumed to carry over successfully, under transformation, to those of the complete operational circuit.

As to the accuracy of characteristics and dynamic behavior required among the varieties of CN, we must immediately distinguish between two classes of sub-networks and elements which may in general be comprised thereby. There is first the more fundamental, or primary structure of the CN, succinctly identifiable as the form the CN would take if everything were perfect, or sufficiently close to perfection. Secondly there are those corrective elements and network appendages which may be required since matters are not ideal. For example, auxiliary circuit branches may be added to assure dynamic stability in the operational loop, such features being typically applied in the feedback path (through the CN, as defined) from the OA output to the nulling terminals. Again, compensating anticipatory paths may be desirable, for example applied (within the CN) between the circuit inputs and the OA input or inputs. The irreducible or minimal capacitance of the necessary shielding may also be included in this seconday class of CN citizenry, as may the electrical properties of nonlinear protective devices of various kinds which may be applied to the OA terminals. These (secondary) portions of the CN may be regarded as residuals and "anti-residuals"; in many cases they will not be important. When they are, the accuracy required in dealing with them is ordinarily of a lower order than that for the basic, or theoretical portion of the CN. Typically 5 to 20 percent will suffice for a knowledge of those portions, although the importance of other qualities of the circuit elements involved should not be neglected as a result. We may now return to the basic remainder of the CN.

Naturally the specifiable accuracy of the primary elements of the CN will be governed by the overall accuracy required of the operational circuit. The relationship may not be one-to-one, however, since a given circuit element may contribute to the composite result in either a diminished or a magnified proportion. Acceptable tolerances for these elements may be on the order of one part in a thousand, but will range from ten times worse to 100 times better than this. These tolerances refer to the accuracy of the principal electrical property of the element, which is important but incomplete. At least as important is the purity of "fidelity" with which the element conforms to its principal property, not only in linearity with respect to current and voltage, but in the exclusion of parasitic properties and perturbations. One must further consider the variations which may occur with temperature, time, and operating stresses; these qualifications or stabilities will be chosen with attention to the rigors of the application. For example if the surrounding conditions are uniform and peaceful, more liberal tolerances may frequently be permitted.

For the CN's of operational circuitry to be discussed in these pages, the linear electric circuit elements are largely limited to resistors and capacitors. Inductors have also been useful on occasion, and will be again, especially if they are developed to stages of fidelity attained for the other two kinds of element. Transformers deserve a similar comment, and may ultimately offer a further design flexibility not characteristic of the three basic (two-ended) kinds of circuit element. Elements which are purposeful and accurate mixtures of those we have cited are not unknown, and may someday become prevalent. Even more exotic elements, such as negative capacitors, may be synthesized by operational feedback, as may be demonstrated. On the whole, however, the linear elements of the CN may be considered as being either resistive or capacitive, and in the ranges from 10^2 to 10^8 ohms

and from 10^{-10} to 10^{-4} farads. In passing we may call attention to the quite generous million-to-one useful ranges over which these elements are available in excellent tolerances, stabilities, and fidelities. It is in fact one of the strengths of the OA technology that it permits exploitation of this availability, applying high performance feedback to ensure a wide range of precise characteristics which are made to depend on those of the above-named electric elements: as contained in the CN.

Static linear operations, the most universal of which amounts algebraically to a linear combination, are ideally obtainable with nothing but orthodox resistors in the CN. Dynamic linear operations – a more general case – require the presence of capacitors in the CN: one or more depending on the complexity of the dynamic operation intended, and usually with resistors included as well. As already implied, these elements may be fixed or adjustable in value. When adjustable, their variation may be continuous or incremental, manual or automatic. A magnificent selection of variable resistors is available. In contrast, capacitors above about 10^{-9} farad are seldom continuously adjusted, especially over wide ranges.

So much, at least for linear elements, in this context. These elements are employed in the majority of CN's simply because the majority of operational circuits are asked to perform linear operations. Even for linear applications, however, the CN may contain nonlinear elements, chiefly as protective auxiliaries. The more general form of CN will employ an interconnected set of linear *and* nonlinear elements, even in the basic or theoretical portion. A CN built exclusively of nonlinear elements is not only a practical possibility, but actually represents an important subclass. In the most general case, for nonlinear dynamic applications, it will be recognized that the CN is an object which presents formidable analytic difficulties, even without inductors or transformers, even when all circuitry is strictly passive, and even when the nonlinear elements have conductive (or resistive) properties alone. Nevertheless the operational technologist need not be alarmed or discouraged, since he is in large measure a synthesizer anyway – and in possession of powerful experimental tools. Parenthetically, we must remind ourselves that these remarks apply in an altered sense when unknown "natural" structures are actually contained in the CN.

The most familiar and broadly useful nonlinear element for the CN is the diode or rectifier, which quite effectively favors flow in one direction over the other, being thus closely analogous to a fluid check valve. Historically, the forms of diode most useful for operational circuitry were thermionic, whereas more recently semiconductor diodes have won out over the thermionic variety in most respects. Being superficially similar in electrical characteristic, these two classes of diodes are different in important details. Curiously, the older diode offers a usable and widely-ranging logarithmic characteristic in the reverse (or weakly conductive) direction, while the modern semiconductor diode offers a similar function in the forward (or strongly conducting) direction. The thermionic form conducts appreciably at zero potential (like a leaky check valve), requiring appreciable reverse potential to reduce the initial current to relatively small magnitude. The semiconductor diode, on the other hand, passes zero current for zero voltage and requires appreciable forward potential before attaining relatively high conductance. Again, the semiconductor diode may have nonlinear and predictable capacitive properties under reverse, zero, and even slightly positive voltages. While most diodes are admittedly far from ideal, they may be persuaded to yield useful nonlinearities for CN characteristics, this being particularly true of certain contemporary varieties. Important cases involving diode-resistor networks will be described later. The application of diodes in counteracting pairs, and the superposition of steady currents from fixed sources, are among the techniques which serve to promote precision. Most of all, however, as compared with the application of diodes in passive networks without feedback, it is the action of the OA itself which can accomplish accurate and predictable nonlinear performance.

The semiconductor transistor itself appears to have advantages over diodes for certain nonlinear applications in the CN. Transistors used in this fashion are actually passive elements. Characteristics like the exponential, logarithmic, hyperbolic, and inverse hyperbolic functions may be obtained in several ways. The triode element is frequently considered superior to the diode form for such purposes. It seems predictable that numerous future applications will be made in CN synthesis of the newer semiconductor devices developed for other purposes.

The Operational Amplifier (OA)

A general discussion of the OA in its own right has been withheld, thus far, in order first to consider the whole operational system or circuit, and within it the co-acting CN, and so to emphasize the vital fact that the OA is functionally dependent on the rest of the local circuit. This local circuit comprising CN plus OA, given adequate power supply, power gain, and stability, will carry out its operations on the CN inputs no matter what happens in the subsequent systems or circuits (or loads) which it manipulates. The nature of this task, briefly, may be computational, regulatory, causally isolating, or any combination of these. To perform this intended function, for various forms of CN and for various and even varying load configurations, the OA must possess a specialized set of characteristics. To present all of these characteristics is not proposed, in this part or even in this series, for they are ramified and in some areas recondite. We cannot hope to instruct in the successful design of an OA, but we do hope to summarize the features which are essential for success.

As exhibited in Figure 1.1 above, the amplifier inputs and outputs amount to energy flows. The power variable at each "port" is, of course, the time rate of flow of energy, or ultimately of electron-volts. Conversely the energy involved in each instance is the time integral of the instantaneous power. This amplifier configuration is seriously incomplete unless we denote an internal supply of power, at one or more sites, to provide the flow of energy required for the amplifier outputs. This internal and absolutely essential power source, as yet incompletely specified, has not been emphasized since it does not involve signals or information in the usual sense. For the sake of completeness it should be added that there are power "sinks" present within the amplifier, as indeed there generally are within the characterizing structure of Figure 1.1 as well. These latter represent dissipation or loss of power, largely in thermal form, and such losses must be taken into account in any assessment of efficiency. Again however, they are not really signals and are not intended to carry information. Of course energy dissipation, in thermal and other forms, is always present no matter what physical medium or combination of media be involved. So nothing unusual in this regard is encountered as we pass over to the OA of Figure 1.2, which we have presented as entirely electric, with power inputs and output in the shape of voltage-current couples. As first stated, we still must imagine power to be supplied internally to the amplifier, this time in electrical form, even though information is carried only in the signals shown.

To recapitulate in part, the OA inputs are shown as involving both voltages and currents, the individual power variables themselves amounting to instantaneous voltage-current products. We may normally assume that power flows *into* the amplifier in each case, although it is dangerous to generalize quite so glibly in these matters (especially within feedback systems!). It is universally true, however, that the power levels involved at the OA inputs are very much smaller than are those at the OA output, at the principal circuit output, or even ordinarily at the operational circuit inputs. The power consumed at the OA inputs is also normally smaller than that involved within the CN.

Further as to the OA input signals, shown as a pair of voltage-current couples in the general case of Figure 1.2, the two couples are traditionally symmetrical: when both exist, that is. If one encounters high impedance within the amplifier, so does the other; if one encounters low impedance, so does the other. These input impedances are of course important properties of the OA, and our assessments are to be taken with respect to the impedances inherent in the CN. It is evident, either in the case of relatively high or relatively low input impedance, that the power drawn by the OA from the CN is bound to be small owing to a gross mismatch. If any input impedance of the OA were comparable to those in the CN which are relevant, and if such impedance were sufficiently well known (both qualitatively and quantitatively), then a satisfactory outcome might still follow if the impedance involved were considered as part of the CN. This consideration is more poignant in the case of pairs of OA inputs than for that of a single input couple – to which we briefly turn.

In the important case of a single OA input couple, the input power is further reduced by the feedback action of the operational circuit, for any given input impedance, since the input voltage or the input current (or both) may be brought close to a null by the loop action. With a pair of input signals, a null is sought only in the

difference between either two input voltages or two input currents; such a null, or equality, does not lead to minimum input power except in special circumstances, as when with a three-terminal amplifier input configuration the differential input impedance or admittance leads to greater dissipation than the common path does. These distinctions usually appear less subtle in practical cases.

It may seem obvious that an OA with a high input impedance responds to voltage, and that one with a low input impedance responds to current. This conclusion must not be arbitrarily accepted, however, although it contains a good deal of truth. Certainly one may take the attitude that, for any given input impedance, the OA responds to both variables, and that the ratio of the two amplifier gains which result, when each kind of variable is regarded as an input, is directly given by that input impedance. The meaning of the question of whether an amplifier responds to voltage or to current is actually more profound, however, than simply how it affects the gain efficiency. It relates indirectly to performance in terms of the signal-to-noise ratio, and sometimes in terms of the speed of operation, for example. As to noise, which mixes perturbations with the input signals, it may contribute additively or multiplicatively thereto, and affects the resolution of which the amplifier is capable, and in turn its usefulness as a precision instrument. There are several sources and kinds of noise, not all well understood; but it is at the OA inputs that its debilitative effects must be most carefully guarded against. In particular, as regards noise sources within the OA, these must at least be classified as to equivalent input voltage noise, as to equivalent input current noise, and as to their frequency spectra and amplitude distributions. Among other important considerations is that of the susceptibility of the OA inputs to inadvertent perturbing signals, random and otherwise, which intrude from the environment.

Consider for the moment only the single-input variety of OA, those accepting one voltage-current couple as input signal. In speaking of its input impedance, perhaps more than of impedances elsewhere, it is best to recognize that these are in general small-signal concepts. It certainly may be, in the case of certain amplifiers, that quite large input signal excursions experience, from their point of view, a constant impedance at the OA input; but in an important and large fraction of cases this is far from true. In such cases it is only possible to deal in fixed impedance ratings because the input signal excursions themselves are maintained at quite small values by the operational feedback action. In any case, when input impedances are cited in terms of resistance, or input admittances in terms of conductance, this should not automatically be interpreted as being independent of frequency. While such citations may be valid (for small signal levels) at DC or at low frequencies, for higher frequencies the equivalent input resistance or inductance figures should be accompanied by the assumed frequency or frequency range, or else an equivalent reactive circuit should appear in the testimony. Even so such data may be valid, as a linear conception, for a limited signal range only. The nonlinear character of an OA input, when signals exceed the normal range of excursion, may alter behavior drastically.

The remarks immediately above, on the single-input OA, apply also to those with a symmetrical pair of inputs when one of the inputs (or terminal pairs, rather) is purposely held at or near zero signal level. This is common practice with differential amplifiers, in fact, and there are many excellent reasons for its popularity.

In the fairly general OA circuit of Figure 1.2, two independent input signals are shown, each carried on a pair of conductors. Identical currents are assumed to flow in the two conductors of each pair, just as in the case of all other signals surrounding and internal to the operational circuit. This luxurious arrangement was quite consciously selected in order to emphasize the energy-flow nature of all signals, and since it covers without too unwieldy a structure substantially all the diverse special cases known to be in use today. Certainly it may not include every departure which will appear in the future, or even those now in development, although it is fondly hoped that most of these will not have slipped through the net. If the reader is aware of familiar instances apparently not encompassed by our general case, we can expect at most a temporary indulgence. One of the familiar OA input arrangements, for example, is that of the direct-coupled differential amplifier, which appears to offer only two input terminals – one for each polarity. This is generally not adequately described by one of the input couples of Figure 1.2 alone, since first there may be a net current

flow into the OA which is thus unaccounted for, and since further the input impedance characteristics are rarely quite symmetrical. Such a case is possibly best accommodated by collapsing the four OA input conductors into three, one then being "common", and by recognizing the possibility of additional common or reference connections, not necessarily involving power, between internal portions of the OA and the CN. See Figure 1.4. Other cases are of course equally important, but this example should suffice for the present.

The reaction of the OA to its separate inputs signals will ordinarily not be precisely additive, or even expressible by a purely algebraic linear combination, except possibly for small signals in the steady state. Each signal will be recognized according to a specific dynamic relationship. The distinction between the two signal inputs will only be unimportant in the event that one is inactive, or when it changes only very slowly, as in the so-called single-ended OA's. As already pointed out, this well known and useful special case may be conveniently served by the more versatile kind of OA as well.

It should now be safe to point out, having gone to some lengths to avoid misinterpretation, that in typical differential OA's the intention is to obtain equal and opposite response to the input signals, and that in many cases this is substantially achieved – or can be brought about by easily understood compensations. In any case the departures from balanced behavior are generally due to the first stage or stages of the amplifier. Better expressed, in a good DOA the signal to which subsequent stages respond is a rather impartial intermingling of the two inputs.

As for the internal structure and function of the OA, between input(s) and ultimate output, only a few completely general remarks may be made. It is somewhat like describing together in a few pages, all fauna superior to the earthworm: in some ways worse. The simile is actually not trivial, though we shall not develop it at length here. Animals are active entities, having sensors, motor facilities, energy sources, and cybernetic feedback relations with environing impedimenta. The varieties of OA evolve more rapidly, though perhaps not as wondrously, and are admittedly somewhat more rudimentary; but their species are still impressively variegated.

The paths of information flow within an OA involve one or more sequences of amplifying stages, each such stage having power gain, through not always voltage or current gain *per se*. At least one such sequence will have DC coupling all the way from input(s) to output. The sequences may join or bifurcate in a variety of ways. The amplifying stages themselves may be balanced (symmetrical) or unbalanced (asymmetrical), but each will be supplied with power from one or more sources or supplies. Normally the amount of power required and expended increases progressively from input to output, culminating in a final stage, which may be contained in a separate booster component, and which is intended to drive the output over a specified maximum range of current or voltage. Of the latter variables, one is operationally demanded by the circuit inputs *via* the CN; the other is then whatever is required by the total load, and the final OA stage must be enabled to furnish the consequent power up to some preassignable limit.

The sequences of amplifier stages in the OA must be supplied with power in such a way that spurious interactions cannot take place between them. As is usual, such power comes from a source or sources shared by numerous stages, and this implies either unimpressionable sources or decoupling compensation and filtering of thoughtful design. Random fluctuations in the power sources must also be prevented from reaching the stages toward the input end of the OA. Such noise and perturbations will be less important in the latter stages owing to the feedback loop, just as in the extreme case of fluctuations in the load.

Aside from the interconnected sequences of amplifying stages which make up the skeleton of the OA, typically there will be a number of coupling connections having a variety of purposes. Included among these are: DC couplings, both feedback and feedforward, to improve DC and low-frequency performance; AC couplings to improve high-frequency performance and to promote both internal and operational loop stability; compensatory biases involving either voltages or currents; and protective features to minimize harmful consequences deriving from incidents or accidents beyond the OA confines.

In the hierarchy of electric networks, the OA is both active and nonlinear. For small signal excursions, however, it may be regarded as

approximately linear, and this is habitual. In this connection, note that it is safest to speak of small *output* excursions, since many amplifiers will violate the linear realm grossly even with quite small input signals, whether these are AC or DC. The linear science of OA dynamics is actually not very profound, and is only arduous when time-wise discontinuities, or some other such complications, are involved. There is nevertheless still some controversy over what constitutes an optimum linear characteristic. For any given application, when the CN and the load are both known, it is relatively clear what OA dynamics will give the desired result – in the linear band of operation. For a *range* of applications, however, it is only clear that a compromise must be accepted. However, it is generally conceded that the most versatile dynamic OA character is that of a time integrator: routinely expressed in terms of signal amplitude versus frequency. It is normally implicit that polarity is reversed through the amplifier, so that the characteristic is referred to the negative input, if there are two.

For a fashionable class of OA, it is common to base the characteristic on its output current relative to its input voltage, so that its amplifying property has the dimension of a conductance – and amounts then to a transconductance. A numerical gain results, of course, once a particular load resistance is specified, providing in this instance a voltage ratio. For power gain, the input impedance of the amplifier must also be known. As to the amplifier's own output impedance, there may be additional reasons for asking that this be substantially lower than that of any typical linear load. Notice that we are talking of static properties here, having already referred to the desirable dynamic property of time-integration, which implies an infinite DC gain by definition. Infinite gain is of course a transparent fiction, just as pure integration is an approximate claim. When infinite gain is attempted, by trickery, the mark may be exceeded accidentally, giving then a large reverse gain over part of the output signal excursion. In some applications, though not all, the result may be an undesired hysteretic behavior. One of the difficulties besetting the striver after infinite gains, or even very large gains, is that the slope of the output-input curve cannot be maintained uniformly over the range of output excursion. Fortunately, more modest gains will almost universally suffice. The power gain for most OA's will in fact lie in the range from 10^8 to 10^{15}, these being naturally the DC figures.

Thus, in particular, a true integrator characteristic is never attained, but at best that of a first-order tardigrade dynamic, which is difficult enough. Such dynamics are substantially as versatile as pure integration, especially as concerns loop stability. On this score, the problem is rather that the more nearly instantaneous portions of the transient response depart from the ideal "integrating" shape owing to one or more cascading delays. Such phenomena may be described in equivalent form by frequency spectra, but the phase data must be attended to diligently along with those for amplitude. For the highest performance these "parasitic" delays must be assiduously minimized in the OA. They irresistably invite loop instability, or at least degrade performance since the principal tardigrade must be slowed down or otherwise moderated, by means internal or external, in relation to the accumulated cascades. Finally, as to linear behavior, let it be emphasized that stability in the linear region is a necessary but not sufficient condition for stable OA operation: every OA is a hotbed of nonlinearity. Without provision for appropriate behavior in the nonlinear condition, temporary or possibly sustained instabilities may result from even a brief adventure into nonlinear territory. We shall look briefly, in a moment, at some of the ways in which the linear realm may be quite innocently transcended.

But first it seems worthy to rephrase one of the special requirements of linear OA dynamics. In contrast for example with most communication amplifiers, in fact with most amplifiers not primarily intended for strongly fedback operations, the introduction of time delays is disastrous. To build a so-called high-fidelity amplifier, or a so-called wide-band amplifier, in many instances, it is considered permissible to sacrifice phase response at higher frequencies to enable diminished distortion. This amounts to introducing delay, which may be of no consequence in open-loop applications, but which is asking for trouble in an OA. Paradoxically, as a class, operational circuits based on OA's produce the least distortion of any continuously-acting electronic information processing means. Ironically, the OA is sometimes referred to as a "low-pass" DC amplifier. It is somewhat more than that!

On the problems of nonlinearity, these arise in a number of different ways, many of which are rather complex, and some of which are arcane indeed. Since excursions in voltage or current or both increase progressively as one passes down the OA stages toward its output, it is to be expected that nonlinear behavior will first be experienced in the final amplifier stage or stages. This is generally the case. Most nonlinear phenomena are indeed observed there. The fewer instances in the earlier stages include interesting examples in which nonlinear characteristics may be purposely applied to advantage to guard against undesirable behaviors.

The most familiar kind of nonlinear OA property is that encountered when an output excursion, whether of voltage or current, tries to go beyond the range which the final stages were designed to provide. This may happen either because the operational input demands, transmitted via the CN, ask for such an excursion, or because the load is too onerous. The next most familiar nonlinear OA event is also evident at the output, and is normally associated with the abilities of the latter stages. This amounts to limiting of the time rate of change of the output voltage or current. Since any given OA will have a specifiable maximum capability in terms of these rates, even under the most favorable kind of load, such limiting is in the first place the result of too steep a demand originating at the operational circuit inputs, interpreted and monitored by the CN. When appreciable reactance appears in the load, such limiting arises under milder demands, and is attributable also to the output limit on voltage or current. It is further evident that limiting behavior in the higher time derivatives can and does occur, in particular, at the OA output, but not only there. Classically, if each stage had a first-order delay, then a limit for the n-th time derivative of the output signal would be associated with a limit of the excursion itself at the n-th amplifying stage counting back from the output, but of course in practice things are not usually that simple. For example, such delaying stages are frequently by-passed by feedforward couplings, and many other corrective means are employed, not only to avoid cumulating delays but to assure stability and optimal recovery when this class of nonlinear behavior is initiated.

When a limit of the sort described takes place at the OA output, it obviously disrupts the continually-acting loop closure through the CN and back to the OA input(s). As a result the nulling or following action at the input or inputs can no longer be maintained by the feedback action. The OA input signal(s) may be observed to depart, either momentarily or for sustained periods, from their nearly-zero or nearly-equal condition. The dynamic or functional transformation prescribed by the CN between the operational inputs and the output will then be violated, at least during the limiting period. Casual isolation may then suffer, along with the rigor of the mathematical, or operational, relationship desired. The input signals to the OA, usually deviating little from a proper null, may encounter abnormal conditions as they depart, and this may either help or hinder the recovery, but the eventuality must be considered. The avoidance of limiting is naturally one of the common goals in specifying the OA capabilities in relation to the signals and load surrounding the operational circuit, but avoidance of the more evil consequences of inadvertent limiting may be achieved by certain non-linear circuitry which may be incorporated in the CN as we have defined it. Such contrivances may of course be regarded alternatively as part of the OA.

One such means, to be described elsewhere, comes into play just beyond the "normal" ranges of OA output excursion. While it cannot prevent output excursions from limiting, for example, it can regularize the behavior and make it predictable. Moreover, by thwarting the disruption of operational feedback action, it may sustain the nulling condition at the OA input, maintain a readiness for instant resumption of orthodox loop behavior after removal of the cause for limiting, and retain the secondary feedback benefits of isolation and low (or high) output impedance.

Of the numerous other classes of nonlinear characteristics which originate in and around the OA, some are more serious than others. An example of unstabilizing nonlinearity which should be avoided is short-term hysteresis in the amplifier's forward path, especially in its direct high-speed paths. Long term hysteresis, or lack of retrace between input and output seen on slow examination, is less alarming since the feedback action will generally have time to circumvent it.

Even hystereses of the faster variety, if unavoidable or desirable for some mysterious reason, may be reduced to harmlessness by negative feedback tactics. This seems just in view of the fact that positive feedback, internally applied, is a common source of the offending debility.

Curvilinear nonlinearities such as are introduced by saturation phenomena in the individual OA stages, unless extreme and near the output, are routinely eliminated or reduced to negligible dimensions, by the operational loop in its standard role. Sufficient gain is traditionally provided to make unimportant the nominal attenuations caused by local slope variations over the operating range. One has only to essay the design of a multi-stage open-loop amplifier of high input-output linearity to learn what a blessing is afforded, in tolerance and forgiveness, by operational feedback. Certainly there are amplifier applications in which such feedbacks are thus far inapplicable; these applications must generally be satisfied with only mildly predictable characteristics (such as linearity). The operational circuit technology is happily not one of these.

Recall that the output impedance of the complete operational circuit is ordinarily dependent on the presence of the OA-CN loop. Typically the output impedance is thereby greatly lowered in the case of a voltage output, and increased in the case of a current output. Nonetheless it must be emphasized that the output impedance of the OA itself may be important for several reasons; the loop must not be expected to bear the whole burden. It is first a case of the loop action augmenting the OA output impedance (or admittance) in a multiplicative manner, so that a high-performance output is made better in proportion to the loop gain. Furthermore there are circumstances, some of which have already been cited, in which the feedback is disrupted, leaving the OA to handle the load on its own. Somewhat less obvious such circumstances arise when the CN, by its very nature and intention, does not provide effective loop action when certain kinds of operational input signals are submitted to it. It may be a great comfort if the OA itself, by virtue of its inherent output capabilities, is then able to cope unaided and creditably with the load.

We have neglected the auxiliary OA output, and so may briefly consider it here. It was included out of a respect for completeness and generality; in most instances it does not exist. An example of a case in which it is useful appears when one wishes automatically or remotely to signal an abnormal behavior. Thus if the successful nulling regimen at the OA inputs is lost, as noted above when the OA output limits, for example, this may elicit a distinctive signal behavior at a point within the OA and manifested at the auxiliary output. Such a special output signal, denoting the abnormality, may then either suggest that the current operational results be regarded with suspicion, or may itself alleviate the condition causing the abnormality.

Since we have considered balanced OA inputs of opposite signs, we might also have provided for balanced OA outputs as well, such as serve to such good advantage in other kinds of amplifier. The reason we have not is fairly explicit. Oppositely acting OA outputs are principally useful, in developments to date, only when the output signals involved are quite precisely mirror images of one another, at least up to the accuracy expected of the principal operational circuit in question. Such performance generally requires what amounts to an auxiliary operational circuit, comprising a distinct OA and a specialized CN. The so-called unity-gain inverter provides the required mirror image of the otherwise single-ended OA output; such single-purpose circuits are described later. Thus we have taken the attitude, at least in the present part, that when two output signals are required from an OA, one being mathematically related to the other, that a separate operational circuit is to be appended. This procedure is in fact standard practice. In this sense, more than one OA being involved, we are dealing with an interconnected pair of operational circuits. Thus this useful case, yielding to the demand for operational outputs of each algebraic sign, is at least formally beyond our immediate scope. Its subsequent treatment may partly atone for the stigma of such narrow propriety (or "buck-passing"!).

The Operational Amplifier Finally Defined

A single definition of the OA which is limited to electrical forms, while in keeping with most of the subject matter here presented, and suitable unpretentious, must certainly become obsolete when and if newer non-electrical forms appear in appreciable number and variety. It

seems certain that such types, embodying for example optical and acoustic signals and media, and in combination possibly with other informational technicalities, will not only emerge but will, by inevitable analogy with today's OA, be called Operational Amplifiers. Thus, in the spirit of our opening paragraphs, we frame the following definition of an OA:

DEFINITION An operational amplifier is a device having power gain down to zero frequency, for use in a feedback loop, which so manipulates an output variable or variables, in response to a local input variable or variables derived from an associated characterizing structure, as to enforce a prescribed functional relationship between one such output variable and a set of variables supplied to that structure, through reducing the local input variable or variables substantially to a null or to equality.

In the restricted electrical case we offer the following, related

DEFINITION An operational amplifier is an active, direct-coupled electrical subsystem having power gain, for use in a feedback circuit, which so manipulates an output voltage or current, in response to local input voltages or currents derived from an associated characterizing network, as to enforce a prescribed functional relationship between a related output voltage or current and a set of currents or voltages supplied to that network, through bringing about a substantial balance or null among those local voltages or currents.

Supplementary Definitive Comments:

(a) The function of the operational amplifier is essentially regulatory;

(b) its purpose may be regulation, simulation, or data-processing;

(c) it is adapted to collaborate with similar amplifiers in adjacent feedback circuits.

Mathematical Summary

We have avoided equations, if not mathematical symbols, completely up to this point. Most quantitative content has resided in the block diagram and circuits, and the attendant descriptions thereof. For those who might be happier with at least some sort of mathematical statement, and for future reference, we append here an incomplete and imperfect collection of such statements about the local operational system and circuit. Please note that among the physical properties *not* embodied by this sort of symbolic means is the very important one of causality.

The power balance for the characterizing structure of Figure 1.1 may be expressed as

$$\sum_{i=1}^{n} (w_i) - w_{PC} - w_{KC} - w_{DC} + w = w' + (w_+ + w_-) \qquad \text{I}$$

where the newly introduced power symbols w_{PC}, w_{KC}, and w_{DC}, represent, respectively, the rates of growth of total potential and kinetic energy, and the total power dissipation. All quantities are instantaneous. Long term averages for w_{PC} and w_{KC} will be zero.

Similarly, for the amplifier in Figure 1.1

$$w_a + w_{PA} + w_{KA} + w_{DA} + w = w_S + (w_+ + w_-) \qquad \text{II}$$

where the rates, the rates of growth of potential and kinetic energy, and the dissipation are defined analogously to those above. Here w_S is the instantaneous power supplied from the sources available to the amplifier.

The relatively minor power quantities, normally, are the amplifier input power ($w_+ + w_-$), the auxiliary output power w_a and of course the long term averages of potential and kinetic rates of energy growth. These latter may of course have quite appreciable instantaneous magnitudes. The w_i input powers and the dissipation

w_{DC} are generally nominal, thus leaving w_S, w, w_{DA}, and the output power w'. These are the principal contributors to long term power flow. System efficiency may obviously be expressed in terms of the averages of these quantities.

The instantaneous power variables of Figure 1.1 are shown as voltage-current couples in the electrical incarnation of Figure 1.2. More generally, they may be regarded as effort-flux couples, and the "signal" may be considered in each instance to be embodied in, and measured by, one of the members of the couple. For example in a mechanical system all signals could be forces, or all velocities, or they could be an assortment of the two; power is still given individually by the effort-flux products. Whichever variable is considered in each instance to embody the signal may be denoted by an arbitrary symbol, with the *caveat* that the physical dimensions will not in general be the same throughout. Thus, in Figure 1.2, any input e_j, or i_j, for which the power is instantaneously $w_j = e_j i_j$, either e_j or i_j may be called v_j, depending on the circumstances and the option. Similarly throughout the operational circuit.

In Figure 1.2 we may recognize a forward path in the CN, from the circuit inputs to the OA inputs, and a feedback path similarly directed but from the OA output. In the simplest static, linear case, we may write,

$$v_+ = \sum_{i=1}^{n} a_i^+ v_i + a^+ v$$

and

$$v_- = \sum_{i=1}^{n} a_i^- v_i + a^- v$$

where each coefficient a is assumed to be resistive, conductive or numerical; the superscript symbology should be self evident. This formulation conceives the amplifier input properties to be appended to the internal outputs of the CN via the nulling links, if indeed the CN characteristics are thereby appreciably altered.

emphasis on the causal cycle involved in the operational circuit, we offer Figure 1.5, which illustrates in summary the interwoven paths of influence and information flow which that circuit entertains.

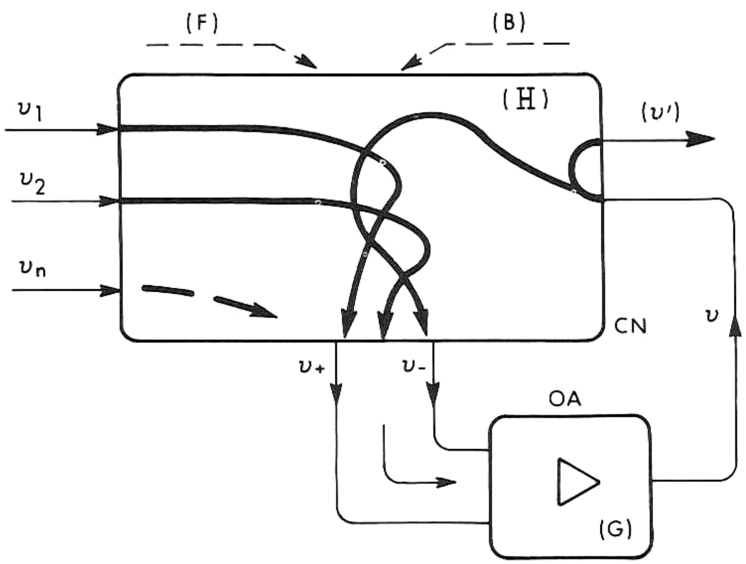

Figure 1.5

Block Diagram Illustrating Circulation of Influence